

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Show that the equation of the curve given by
 $x = ae^u \cos u$, $y = ae^u \sin u$, $z = be^u$ are

$$K = \frac{\sqrt{2}a}{(2a^2 + b^2)^{1/2}} \cdot \frac{1}{s}, \quad \tau = \frac{b}{(2a^2 + b^2)^{1/2}} \cdot \frac{1}{s}.$$

17. (a) Find the angle between the two directions on a surface at a point P having directions (l, m) and (l, m) .
 (b) Show that curve $du^2 - (u^2 + a^2)dv^2 = 0$ on the right helicoids form on orthogonal set.
18. State and prove Gauss-Bonnet theorem.
19. Derive the Rodrigue's formula.
20. State and prove Hilbert's lemma.



APRIL/MAY 2025

23PMA42 — DIFFERENTIAL GEOMETRY

Time : Three hours

Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Explain function of class m .
2. Define osculating plane.
3. What is metric?
4. Define surface of revolution.
5. Prove that $U\ddot{u} + V\ddot{v} = \frac{dT}{dt}$.
6. Define Geodesic.
7. State Euler's theorem.
8. Define Dupin's indicatrix.
9. State Hilbert's Lemma.
10. Define complete surfaces.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Find the equations of the circular helix $r = (a \cos u, a \sin u, bu)$ $-\infty < u < \infty$ where $a > 0$, referred to s as parameter and show that the length of one complete turn of the helix is $2\pi c$, where $C = \sqrt{a^2 + b^2}$.

Or

- (b) Calculate the curvature and torsion of the cubic curve $\vec{r} = (u, u^2, u^3)$.
12. (a) Prove that if θ is the angle between the two direction at (u, v) on the two family of curves. $pdu^2 + 2Qdudv + Rdv^2 = 0$ then

$$\tan \theta = \frac{2H(Q^2 - PR)^{1/2}}{ER \cdot 2FQ + GP}$$

Or

- (b) Find the coefficients of the direction which makes an angle $\pi/2$ with the direction whose coefficients are (d, m) .

13. (a) State and prove Liouville's formula.

Or

- (b) Prove that if (λ, μ) is the geodesic curvature than $kg = \frac{H\lambda}{Fu' + Gv'} = \frac{H\mu}{Eu' + Fv'}$.

14. (a) Prove that the tangent to the edge of regression are characteristic line.

Or

- (b) State and prove Euler's theorem.

15. (a) Show that the only compact surface whose gaussian curvature is positive and mean curvature const are sphere.

Or

- (b) Prove that if at all points of a geodesic the Gaussian curvature is less than $1/b^2$, the curve is necessary of shorter length than neighbouring curves along an arc length at least equal to πb .

